MAT126 Homework 3

Problem 1. After performing a long division, evaluate the integral

$$\int \frac{x^5 + 4x^3 + 2x^2 + 1}{x^2 + 1}\, dx.$$

Problem 2. Consider the integral

$$\int \frac{x + 2}{x^3 - 6x^2 + 9x}\, dx.$$

(a) Write the integrand in its partial fraction form, and find the resulting constants A, B and C.

(b) Evaluate the integral.

Problem 3. Consider the integral

$$\int_0^{3/2} x \cos(x^2)\, dx.$$

(a) Approximate the integral using the mid-point rule with $n = 5$. Give your answer to 4 decimal places.

(b) Approximate the integral using the trapezoidal rule with $n = 15$. Give your answer to 4 decimal places.

(c) Denote the error in your approximations by E_M and E_T, respectively. By first finding a bound for the second derivative of $f(x) = x \cos(x^2)$, use the error bounds we have seen in class to bound $|E_M|$ and $|E_T|$. Leave your answers in exact form.

(d) What value of n should we choose to guarantee that both the mid-point rule and trapezoidal rule will have error less than 0.01?

Problem 4. For each of the following integrals, explain why they are improper integrals. Then, evaluate them. If they are divergent, then write 'DIVERGENT'.

(a)
$$\int_0^1 \frac{1}{\sqrt{x}} \, dx.$$

(b)
$$\int_2^4 \frac{2x}{x^2 - 9} \, dx.$$

(c)
$$\int_0^\infty (1 - x)e^{-x} \, dx$$

Hint: Proceed with integration by parts, and make use of the limit
$$\lim_{t \to \infty} te^{-t} = 0.$$

(d)
$$\int_0^{\pi/2} \tan\theta \, d\theta.$$

Hint: Write $\tan\theta$ as a fraction involving $\sin\theta$ and $\cos\theta$, and use a substitution.