MAT126 Homework 2

Problem 1. Evaluate the following integrals:

(a)
$$\int_3^5 \frac{1}{x+3}\, dx.$$

(b)
$$\int e^x \sin(x)\, dx.$$

(c)
$$\int \tan^4 x \sec^6 x\, dx.$$

(d)
$$\int \frac{8x-1}{5x^2+2x-3}.$$

Problem 2. Let $f(x)$ be any differentiable function. Prove the below formula, using a substitution.

$$\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c.$$

Problem 3. Consider the integral
$$\int \frac{x+6}{4x^3-7x^2-15x}\, dx.$$

Solve it by writing the integrand (this means the function that is being integrated) in the form

$$\frac{A}{x} + \frac{B}{x-3} + \frac{C}{4x+5}.$$

Problem 4. Consider the integral

$$\int \frac{x^3}{\sqrt{x^2 + 9}}\, dx.$$

(a) According to lectures, which substitution should we try if we see the term $\sqrt{x^2 + a^2}$?
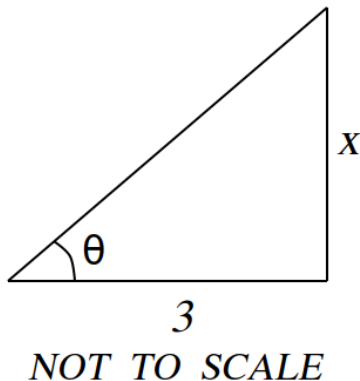
(b) Make a substitution to show that our integral is equal to

$$27 \int \tan^3 \theta \sec \theta\, d\theta.$$

(c) Using another substitution, show that

$$27 \int \tan^3 \theta \sec \theta\, d\theta = 9 \sec^3 \theta - 27 \sec \theta + c.$$

(d) Suppose we have the following triangle:

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Using SOH CAH TOA, what is the value of $\tan \theta$? What is the value of the hypotenuse? What is the value of $\cos \theta$? What is the value of $\sec \theta$?

(e) Evaluate the integral given at the start of the question.