MAT126 Homework 1

Problem 1. Define the function $f(x)$ by

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 1 \\ x & \text{if } x > 1. \end{cases}$$

(a) Using a theorem from class, explain how we know that f is integrable on $[0, 3]$.

(b) Evaluate the definite integral $\int_0^3 f(x)\,dx$.

Problem 2. Consider the below definite integral:

$$\int_3^5 \frac{1}{x+3}\,dx.$$

(a) Using left endpoints and 4 subintervals, estimate the value of the integral.

(b) Without computing the exact value of the integral, can we say whether our answer to part (a) is an over-estimate or an under-estimate? Why?

Problem 3. You have landed an internship at NASA. It is the day of a big launch, and it is your job to track the height above the earth's surface of the spacecraft. 1 minute after lift-off, all data readings are malfunctioning except for the velocity of the ship at 10 second intervals. Use the below table to estimate the height of the ship after 1 minute.

Time (s)	0	10	20	30	40	50	60
Velocity (m/s)	0	12	40	65	84	72	91

Problem 4. Let $g(t) = (2t+1)^{1/3}$.

(a) Find an anti-derivative $G(t)$ of $g(t)$.

(b) Evaluate the definite integral

$$\int_{-\frac{1}{2}}^{2} g(t)\,dt.$$

Problem 5. A function $f(x)$ is said to be *odd* if $f(-x) = -f(x)$ for all x.

(a) Say whether or not each of the following functions are odd:

(i) $f(x) = 3x^3 + 5x$.

(ii) $g(x) = \cos(x+2)$.

(iii) $h(x) = 2\sin(x)$.

(b) Give another example of an odd function, and draw a graph of your example function on the interval $[-5, 5]$.

(c) Let $f(x)$ be an arbitrary odd function. What is the value of the below definite integral?

$$\int_{-1000000}^{1000000} f(x)\,dx.$$